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The Plastic Yield Behaviour of Polymethylmethacrylate

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The yield behaviour of an amorphous glassy polymer has been investigated under a system of combined stress in an attempt to define a criterion for yield. Sheets of polymethylmethacrylate were compressed in plane strain and the compressive yield stress was determined as a function of the tension applied in the plane of the sheet. The compressive yield stress was found to decrease with applied tension more rapidly than would be expected if the shear yield stress of the material were independent of pressure. The results have been analysed in terms of a Coulomb yield criterion where the shear yield stress is expressed as a constant plus a friction term proportional to the pressure on the shear plane. Birefringent shear zones were observed in the deformed region after the load was removed and these zones were inclined at 52.9° to the plane of the sheet. It was found that if the stresses at yield were expressed as nominal stresses then the inclination of the shear planes predicted by the yield stress data coincided with the observed inclination. It also appears that it may be possible to define a fracture criterion in terms of the applied stress system.

1. Introduction

Plastic yielding in metals is moderately well understood. Yielding takes place in metal single crystals by slip on well-defined crystallographic planes by dislocation movement. The critical shear stress to induce yielding is a well-defined parameter, and is found to be independent of the pressure applied normal to the slip plane (for example, see [1]). In polycrystalline metals the situation is more complicated because co-operative deformation of adjacent grains is necessary, and slip on a number of slip systems in each grain must occur, but the general features of the unit process are preserved. Deformation takes place in shear when the shear stress in any direction reaches a critical value (Tresca yield criterion) or when the elastic shear strain energy density reaches a critical value (von Mises yield criterion). On both these criteria the critical shear stress for yield is independent of the pressure normal to the plane in which yield is occurring.

Plastic yielding in amorphous glassy polymers is not so well understood. The general mechanism by which it occurs is likely to be more complicated than is the case for metals. For polymers there is no known simple mechanism, analogous to dislocation movement, by which deformation can occur. In general terms deformation is likely to involve the bending and stretching of molecular chains, some slipping of chains relative to each other, and possibly the breaking of chains. Deformation of metals takes place at constant volume, and this limitation is implicit in the unit deformation process. For polymers it is not immediately apparent that this must be so, although experimentally it is observed that volume changes during deformation are small, at most a few percent, so that in practice deformation occurs essentially at constant volume.

Since the processes by which an amorphous polymer deforms are complicated, it would be surprising if the critical shear stress to produce deformation was a simple well-defined parameter, and in particular it would be surprising if the critical shear stress was completely independent of pressure.

Various studies have been made of yielding in polymers but relatively little attention has been given to determining the type of yield criterion that can most appropriately be used to describe their behaviour. Thorkildsen [2] deformed thin-walled polymethylmethacrylate tubes using a combination of twisting and internal pressure and found that they obeyed von Mises yield criterion, although he took the yield stress to be the stress to produce 0.2% strain and it is not certain that this strain corresponded to the onset of plastic deformation. Onaran and Findley [3] assumed a von Mises criterion in their analysis of the creep behaviour of rigid polyvinyl chloride under combined stresses.

It may be that polymers obey a von Mises yield criterion in general, but since their basic deformation mechanism is different from that of metals it must be for a different reason, and perhaps a reason that is not valid over a wide range of applied stresses.

2. Experimental

2.1. Experimental Arrangement

In the work to be described, the plane strain compression test as developed by Ford [4] was chosen, with in addition a tension applied perpendicular to the direction of compression. The experimental arrangement is shown diagrammatically in fig. 1. The advantage of such an arrangement is that deformation must always occur in plane strain in the plane defined by the directions of the applied stresses, σ_1, σ_2. Any deformation in a direction out of this plane is prevented by the constraint imposed by the regions of the specimen outside the dies. If frictional forces can be kept small, the stress system is simple to analyse. Compression testing has the additional advantages that the load-bearing area remains constant so that no instability can develop, as occurs with the tensile test, and that it is possible to obtain ductile behaviour from materials such as polymethylmethacrylate and epoxy resins which normally break in a brittle manner in tension. Also it is possible after a test to cut sections from the material between the dies and to look at structural changes in the deformed region.

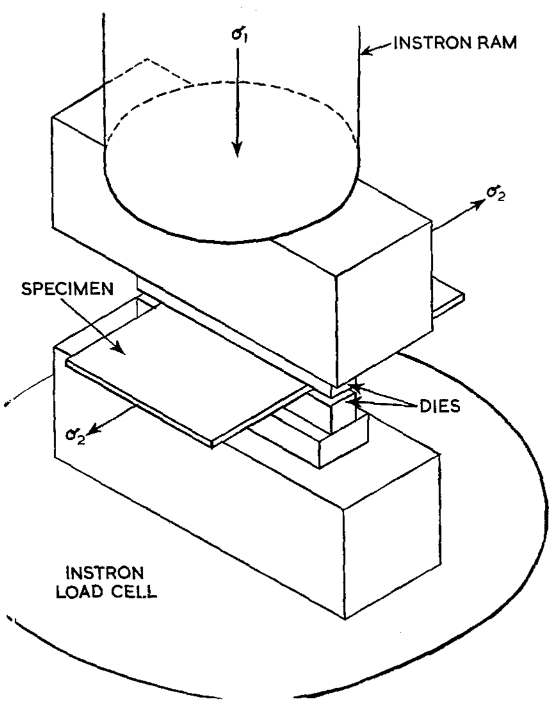


Figure 1 The experimental arrangement.

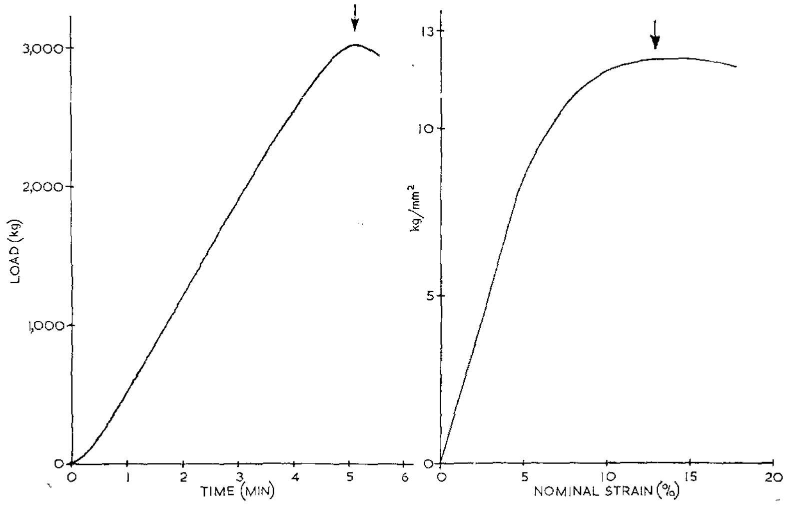
2.2. Experimental Procedure

Specimens of commercial ICI "Perspex" acrylic sheet $\frac{1}{16}$ in. (1.0 in. = 2.5 cm) thick and $1\frac{1}{2}$ in. wide were compressed between $\frac{1}{4}$ in. dies. The dies had slightly rounded edges and had been case hardened and ground flat and parallel to ± 0.0005 in., and subsequently polished. The compressive load was applied with a 5000 kg capacity Instron testing machine, and the compressive displacement in the specimen was measured with a pair of dial gauges reading to 0.0001 in. mounted on the ends of the dies. The tensile load was applied using a small hydraulic ram arranged to force grips on the ends of the specimen apart, and the whole tensile loading system was suspended on soft springs in neutral equilibrium so that the system was self-aligning. The tensile load was measured by monitoring the pressure in the hydraulic system, the force exerted by the ram having previously been calibrated against pressure using the Instron load cell. The dies were lubricated with molyslip grease [4]. The procedure used was to apply a tensile load to some predetermined value and then, holding the tensile load constant, to load the specimen to yield in compression at a constant cross-head speed of 0.02 cm/min. All tests were carried out at room temperature, 22 ± 1° C.

3. Results

3.1. Stress/Strain Measurements

Fig. 2 shows the type of stress/strain curve obtained. In all cases there was a pronounced yield-point, followed by a load drop. The "yield stress" was taken as the maximum value of the



(a) (b)

Figure 2 (a) Instron recording. Load against time at a cross-head speed of 0.02 cm/min. (b) True stress/nominal strain curve derived from (a) by using the observed dial gauge readings to obtain the strain in the specimen.

stress as indicated by the arrows. The time to yield was approximately 5 min.

Fig. 3 shows the variation of the compressive yield stress with applied tensile stress. The compressive stress was obtained by dividing the load by the area of the dies (true stress). The tensile stress was obtained by dividing the tensile load by the initial cross-sectional area (nominal stress). The compressive yield stress is seen to decrease linearly with applied tensile stress, until at high tensile stresses brittle fracture intervenes before ductile yielding can occur. For all points plotted as a full circle (ductile yield) a stress/strain curve of the type shown in fig. 2 was obtained. For points plotted as a cross, the specimen fractured without yielding plastically. The best straight line through the experimental points for which yielding occurred is given by the relation $\sigma_1 = -12.05 + 1.67\,\sigma_2$ kg/mm^2.

3.2. Structural Observations

For a number of specimens the test was discontinued near the yield-point, and thin sections were cut and examined in polarised light. Fig. 4 shows the type of structure observed. It appears that yielding was not homogeneous, but was confined to more or less well-defined shear zones. Fig. 4a shows a section from a polymethylmethacrylate specimen that had been taken just to yield. Fig. 4b shows a similar specimen that had had a number of fine scratches made in the surface, which had the effect of localising the shear zones and making them more distinct, but which did not significantly affect the measured value of the yield stress. Fig. 4c shows an epoxy resin* that had been subjected to similar treatment. Here the shear zones are better defined.

It was confirmed that the "yield stress" corresponded to the stress at which these zones formed in the central region of the specimen. When the test was stopped at a nominal strain of 12.5 %, no zones were found to have formed in the central region. At the yield-point (around 13 % strain) yield zones were present.

From measurements on pictures of the type shown in fig. 4b the mean angle between the shear zones and the plane of the sheet was found to be $52.9 \pm 2.0°$.

3.3. Birefringence Measurements

Birefringence measurements showed that the specimens were initially relatively isotropic. The birefrigence measured in a direction in the plane of the sheet was less than 3×10^{-5}. The birefringence of perspex has been measured as a function of strain [5], and this figure corresponds to a strain of less than 0.5 %, which is small compared with the elastic strain at yield. The birefringence in the shear zones was found to be of the order of 10^{-3}, which corresponds to a strain approximately equal to the elastic strain at yield.

*CIBA MY750 with 20 % plasticiser DYO40.

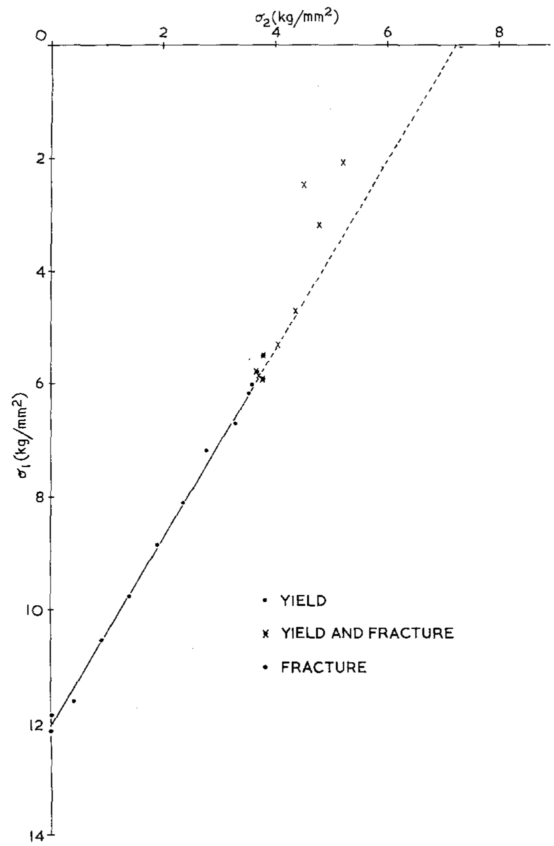


Figure 3 Measured values of the compressive yield stress, σ_1 (true stress) plotted against applied tensile stress, σ_2 (nominal stress). The full circles denote ductile yield, the crosses brittle fracture, and the combined points test where ductile yielding occurred, followed immediately by brittle fracture.

4. Analysis

4.1. General Analysis

The results have been analysed using Mohr's circle construction, from which it is possible to find the state of stress on any plane in the material for given applied stresses. This construction is essentially a plot of shear stress against normal stress. The principal stresses σ_1, σ_2 (σ_1 negative) are plotted on the normal stress axis, and a circle of diameter $(\sigma_2 - \sigma_1)$ is drawn through these points. A point on this circle making an angle 2α with the normal-stress axis represents the state of stress (shear stress and normal stress) on a plane in the material inclined at an angle α.

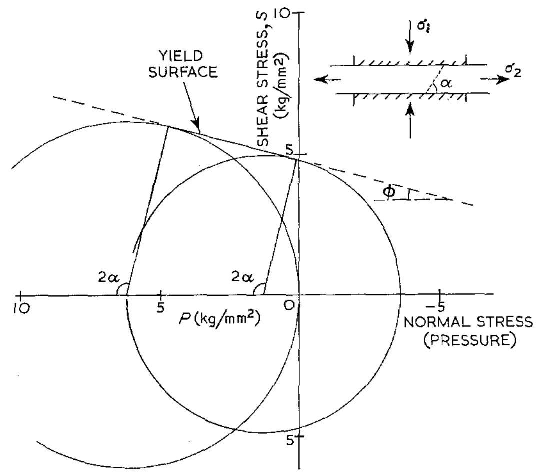


Figure 5 Mohr's circle diagram constructed from the data in fig. 3. The angle ϕ is the angle of internal friction equal to $\tan^{-1}\mu$.

The data in fig. 3 have been plotted in fig. 5. Each point for which ductile yielding occurred defines a circle on the Mohr's circle diagram, and the envelope of these circles defines the straight line labelled "yield surface" in the figure.

This behaviour is consistent with Coulomb's yield criterion [6, 7], which was formulated by Coulomb in 1773 to describe the fracture of building stone and the yielding of soils. On this

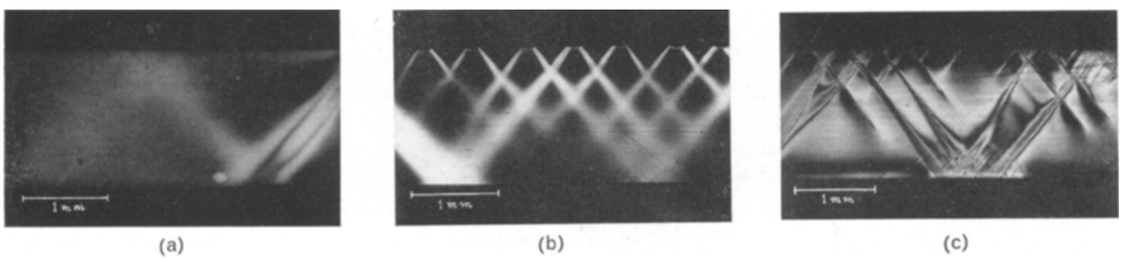


Figure 4 (a) Section of a polymethylmethacrylate specimen, that had been taken just to yield, examined between crossed polars. A pronounced shear zone has formed starting from one edge of the die, but yielding is only just starting in the central region of the specimen underneath the die. (b) As (a), but with scratches made on the surface before testing. (c) Plasticised epoxy resin as (b).

criterion, the critical shear stress for yielding to occur on any plane in the material increases linearly with the pressure applied normal to the plane. The critical shear stress can be written

$$S = k_0 + \mu P$$

where k_0 is the "cohesion" of the material, μ is the coefficient of internal friction, and P is the normal pressure on the shear plane. For the data in fig. 3, $S = 4.66 + 0.258\,P$ kg/mm². (To obtain this relation, uncorrected values of σ_1 and σ_2 have been used. For the fully corrected relation, see 4.4.2.)

According to Coulomb's criterion at the yield-point the stress on some plane in the material has reached the critical value for yield, and the inclination of this plane is defined by the point at which the circle touches the yield surface. For the above yield stress data the predicted value of α is 52.25°, which is close to the measured value for the angle between the observed shear zones and the plane of the sheet of 52.9°.

For an exact analysis it is necessary to take into account a number of corrections that are considered below.

4.2. The Effect of Friction between Specimen and Dies

Although the dies were well lubricated it is possible that the friction between specimen and dies was sufficiently large to exert an appreciable restraint on the specimen and to lead to an increase in pressure under the central region of the dies. The magnitude of this effect can be estimated, and it can be shown that it is likely to lead to only a small correction.

In fig. 6 the forces acting on an element of material between the dies of thickness dx at a distance x from the centre line are shown. P is the pressure acting on this element and μ_s is the

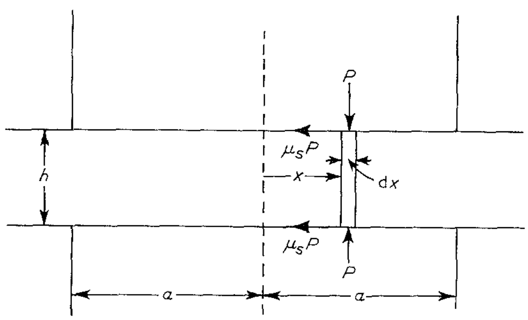


Figure 6 Forces acting on an element of material between the dies. The die breadth is $2a$ and the specimen thickness is h.

coefficient of friction between the specimen and the die surfaces. Resolving horizontally, $h d\sigma_2 = 2\,\mu_s P\, dx$. Yield will occur on the Coulomb yield criterion when $P = (2\,k_0 \tan\alpha - \sigma_2 \tan^2\alpha)$. Hence $dP/d\sigma_2 = -\tan^2\alpha$, and $dP/P = 2\,\mu_s \tan^2\alpha\, dx/h$. Integrating over the surface of the die:

$$P = [2k_0 \tan\alpha - \sigma_2 \tan^2\alpha] \exp [(2\mu_s \tan^2\alpha(a - x))/h]$$

For small coefficients of friction the pressure underneath the dies rises linearly from a value of $[2k_0 \tan\alpha - \sigma_2 \tan^2\alpha]$ at the outside, to a value of $[2k_0 \tan\alpha - \sigma_2 \tan^2\alpha] \times [1 + (2\mu_s a \tan^2\alpha)/h]$ in the centre. The net fractional increase in the measured value of σ_1 due to friction will then be $(\mu_s a \tan^2\alpha/h)$, and this increase will be independent of the value of σ_2.

It has been shown [8] that in a similar type of test with identical lubrication, the coefficient of friction was less than 0.005. Using this value for μ_s, we get a fractional increase in σ_1, of 0.034, or 3.4%. This is a small correction, and is of the order of the experimental error. Inserting the correction will lead to a small reduction in the value of k_0, and also in the value for the coefficient of internal friction.

4.3. Edge Effects

The region of the specimen just outside the dies will exert a restraining force on the region being compressed, and will again lead to higher values of σ_1. It has been shown [4, 8] that this restraining force is independent of die breadth and can be corrected for by making measurements using dies of different breadths.

In these experiments the tests using $\frac{1}{4}$ in. dies were duplicated using $\frac{1}{8}$ in. dies so that the restraining force could be eliminated by subtraction, and the relation between σ_1 and σ_2 in the absence of edge effects could be obtained. This corrected relation is

$$\sigma_1 = -11.10 + 1.54\,\sigma_2 \text{ kg/mm}^2$$

4.4. Elastic Effects

In the stress/strain curve in fig. 2b it can be seen that at the "yield-point" the nominal strain is $\epsilon_1 = -0.13$. This is a very large elastic strain, and affects both the true value of σ_2 at yield, and also the true value of α.

4.4.1. The Value of α

The inclination of the planes on which yielding takes place will be less at the instant of yield than when measurements are made on a section cut from the specimen after elastic recovery. The permanent deformation in the specimen measured after removing the load was found to be less than 1% so the 13% deformation at yield is almost wholly elastic. If Poisson's ratio for the material is taken as 0.4, then at yield the elastic strain in the direction of σ_2 can be calculated to be $\epsilon_2 = 0.081$. If the inclination of the planes measured after unloading is α, then their inclination at the instant of yield, α', is given by the relation

$$\tan \alpha' = \frac{(1 + \epsilon_1)}{(1 + \epsilon_2)} \cdot \tan \alpha$$

For $\alpha = 52.9°$ (measured value), $\epsilon_1 = -0.13$, and $\epsilon_2 = 0.081$, the derived value of α' is 46.0°. So at the instant of yield the inclination of the plane on which yielding is observed to take place is 46.0°.

4.4.2. Definition of Stress – True Stress and Nominal Stress

The area of the dies in contact with the specimen remains constant, and the true compressive stress σ_1, which is the load divided by this area, is not affected by the strain. However, the effective area over which σ_2 is applied decreases in proportion to the strain, so that the true-stress value of σ_2 at yield will be 13% larger than the nominal-stress value.

Using the values of ϵ_1, ϵ_2, given in 4.4.1, and the corrected relation given in 4.3, it is possible to derive the relation between σ_1 and σ_2 at yield, when both σ_1 and σ_2 are expressed either in terms of *true stress* (load per unit area at yield) or *nominal stress* (load at yield per unit initial area). In table I these two relations are given, together

TABLE I

	Compressive yield stress (kg/mm²)	α	ϕ	μ	k_0 (kg/mm²)
σ_1, σ_2 expressed as *true stresses*	$\sigma_1 = -11.10 + 1.365\,\sigma_2$	49.5°	9.0°	0.158	4.74
σ_1, σ_2 expressed as *nominal stresses* "stress per molecule"	$\sigma_1 = -12.00 + 1.667\,\sigma_2$	52.2°	14.4°	0.256	4.65

with predicted values of the angle of friction, ϕ, the coefficient of internal friction $\mu = \tan \phi$, and the "cohesion", k_0.

4.5. The Effect of Strain Rate

Preliminary results have been obtained using a cross-head speed of 0.05 cm/min, which reduces the time to yield by a factor of just over two, and it appears that at higher strain rates both the coefficient of internal friction and the cohesion of the material are increased. The expression for the shear yield stress becomes

$$S = 5.2 + 0.29\, P \text{ kg/mm}^2$$

which should be compared with the uncorrected relation given in 4.1.

5. Discussion

5.1. Interpretation of Results

From the above account, it is evident that elastic effects complicate an interpretation based on Coulomb's yield criterion. Since the inclination of the planes on which yield occurs is, at the instant of yield, very close to 45°, it is tempting to try to explain the results in terms of a Tresca or von Mises criterion (critical shear stress independent of pressure) for both of which one would expect yield to occur on 45° planes where the shear stress is a maximum. However, such an explanation is not consistent with the data presented in table I, since if a Tresca or von Mises criterion is obeyed σ_1 should equal a constant plus σ_2.

Moreover, under these large strains, the structure of the material will no longer be isotropic. If the shear yield stress of the material is anisotropic, the applied stress system will not be the only factor determining the plane on which yield shall take place. Only if the material is isotropic and stress is the sole determining factor can the observation that yield takes place on 45° planes be interpreted as indicating that the shear yield stress of the material is independent of pressure.

An alternative approach is to consider the material to be obeying a pseudo-Coulomb yield criterion. The nominal compressive yield stress varies with nominal tensile stress in a manner consistent with the "nominal" inclination of the plane on which yielding occurs (the value of α measured after the load is removed). It should be pointed out that in this context the nominal stress can have a real interpretation since it is proportional to the "stress per molecule", and is independent of the strain. The true stress has a less specific interpretation since it is the stress per unit area, and the number of molecules per unit area changes with strain. It can be seen from table I that if σ_1 and σ_2 are expressed as nominal stresses, then the predicted value of α of 52.2° coincides almost exactly with the measured angle of 52.9°.

One result of this approach is that it appears to predict a fracture criterion for polymethylmethacrylate. In fig. 5 it can be seen that the largest tensile stress for which ductile yielding occurred is that for which the nominal pressure on the nominal yield plane is zero. For larger values of σ_2, where the pressure changes to a tension, the specimen broke in a brittle manner before ductile yielding occurred. Under these limiting conditions there is a true pressure of 1.1 kg/mm^2 on the plane on which yield is occurring and it does not appear that a fracture criterion can be defined in terms of true stress.

The general conclusion from this work, that in polymers the shear yield stress is pressure-dependent, is consistent with observations that have been made in the yield behaviour and tensile yield stress of specimens cut from oriented polythene sheet [9]. For this material it was found that shear was localised on a plane close to the orientation direction, and that as the orientation direction was rotated relative to the tensile axis, and the tension normal to the shear plane increased, the shear yield stress in the plane decreased.

5.2. The Coulomb Criterion and Plasticity Theory

A yield criterion is a criterion for the onset of plastic deformation (the limit of elasticity), the criterion being defined in terms of the applied stresses. It can be represented by a surface in principle stress space. For an isotropic material any surface that is symmetrical in the three principal stresses can be a yield criterion. In plane strain deformation there will be a functional dependence of σ_3 (the stress perpendicular to the plane) on σ_1 and σ_2, and the surface will degenerate to boundaries in σ_1, σ_2 space.

In this paper the yield-point has been defined as the point at which non-recoverable, and therefore plastic, strains are produced in the specimen, and the criterion for yielding in plane strain has been determined in terms of the applied stresses. Moreover, this criterion can be specified not only as a function of σ_1 and σ_2, but also as a pressure-dependent shear yield stress. The criterion can be stated: *the material will yield when the shear stress on any plane reaches a value given by* $S = k_0 + \mu P$, and yield will occur in shear on this plane. This is the form in which Coulomb formulated his criterion. It does not follow that the criterion in the form just stated will also apply to a general deformation (non-plane strain).

The general theory of plasticity [10] assumes that the material possesses, in addition to a yield criterion, a plastic potential which is a scalar function of stress, such that the ratios of the components of plastic strain rate in particular directions can be obtained from the ratios of the partial derivatives of the plastic potential in these directions. The plastic potential is further assumed to be a surface in principal stress space of the same shape as the yield criterion. This assumption leads to St Venant's principle that the principal axes of stress and plastic strain rate must coincide, and, since the plastic potential is everywhere parallel to the yield surface, to the result that yielding will always occur in a direction perpendicular to the yield surface. If a constant volume condition is imposed it follows that the shear yield stress must be independent of pressure. If the shear yield stress is pressure-dependent, then any shear deformation must be associated with a volume expansion. Deformation in an ideal plastic material is a conservative process, and the volume expansion is necessary to absorb energy that would otherwise be released. Deformation of a material where the pressure dependence of the shear stress is due to friction is essentially a non-conservative process.

For materials which exhibit internal friction and yet deform at constant volume, it is necessary to abandon the concept of a plastic potential. It has been shown [11] that for such materials the principal axes of the strain-rate tensor can deviate by up to $\pm\phi$ from the principal axes of stress, and therefore that the St Venant relations can no longer be applied, even though the material is isotropic. Consequently, many of the relations derived in general plasticity theory are not applicable to such a material.

5.3. The Uses and Limitations of the Pseudo-Coulomb Concept

The pseudo-Coulomb yield criterion allows the states of stress at which yielding will occur in plane strain to be simply defined. However, its practical application to engineering problems is limited at present (apart from the plane strain limitation) because polymers are viscoelastic materials and the criterion only holds for a standard loading path and a standard rate of loading. It is a criterion for yield only, and can say nothing about the behaviour at large plastic strains.

The criterion may be of some help in trying to understand both yield and fracture at the molecular level. Since it is expressed in terms of nominal stress, which is proportional to the stress per molecule, it can be directly related to the molecular processes occurring at yield.

This report has been confined to the behaviour of polymethylmethacrylate, but work now in progress shows that polystyrene, amorphous polyethylene terepthalate, and rigid polyvinyl chloride behave similarly.

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References

1. A. H. COTTRELL, "Dislocations and Plastic Flow in Crystals" (Oxford University Press, Oxford, 1953).
2. R. L. THORKILDSEN, "Engineering Design for Plastics", edited by E. Baer (Reinhold, New York, 1964) p. 277.
3. K. ONARAN and W. N. FINDLEY, *Polymer Eng. Sci.* **5** (1965) 213.
4. J. G. WILLIAMS and H. FORD, *J. Mech. Eng. Sci.* **6** (1964) 405.
5. H. KOLSKY, *Trans. Soc. Glass Tech.* **36** (1952) 56.
6. C. A. COULOMB, *Mém. Math. et Phys.* **7** (1773) 343.
7. K. TERZAGHI and R. B. PECK, "Soil Mechanics in Engineering Practice" (Chapman and Hall, London, 1948).
8. J. G. WILLIAMS, *Trans. J. Plastics Inst.* **35** (1967) 505.
9. A. KELLER and J. G. RIDER, *J. Matls. Sci.* **1** (1966) 389.
10. R. HILL, "The Mathematical Theory of Plasticity" (Oxford University Press, Oxford, 1950).
11. G. DE JOSSELIN DE JONG, Thesis: "Statics and Kinematics in the Failable Zone of a Granular Material" (Uitgeverij Waltman, Delft, 1959).